Centre Cournot pour la Recherche en Économie

Huitième conférence: Augustin Cournot et les modéles de l'économie

Cournot's Principle as a Hypothesis of Market Efficiency

Glenn Shafer

Rutgers Business School

December 1, 2005

Cournot's principle, which gives probability its empirical meaning, can also serve as a hypothesis about the speculative effectiveness of a market. See also Working Paper #15 at www.probabilityandfinance.com.

Outline

1. Inventing Cournot's principle: Bernoulli, Borel, Kolmogorov
2. Making it game-theoretic: Ville, Vovk
3. Defensive forecasting & speculative efficiency
4. The $\sqrt{dt}$ effect
5. The wrong path: Doob, Samuelson
6. The confusion: Malkiel, Shiller

Jakob Bernoulli
(1654–1705)

"A thing is *morally certain* if its probability is so close to certainty that the shortfall is imperceptible."

"A thing is *morally impossible* if its probability is no more than the amount by which moral certainty falls short of complete certainty."

Example: It is morally certain that the frequency will approximate probability in a large number of trials.

How Bernoulli connected probability with the world:

"*Because it is only rarely possible to obtain full certainty, necessity and custom demand that what is merely morally certain be taken as certain.*"

"It would therefore be useful if fixed limits were set for moral certainty by the authority of the magistracy—if it were determined, that is to say, whether 99/100 certainty is sufficient or 999/1000 is required. . ."

Antoine Cournot
1801–1877

Maurice Fréchet proposed the name *Cournot's principle*.

Cournot discussed both

- *moral impossibility* (very small probability) and
- *physical impossibility* (infinitely small probability).

"A physically impossible event is one whose probability is infinitely small. This remark alone gives substance—an objective and phenomenological value—to the mathematical theory of probability."

"L'événement physiquement impossible est donc celui dont la probabilité mathématique est infiniment petite; et cette seule remarque donne une consistance, une valeur objective et phénoménale à la théorie de la probabilité mathématique."

Émile Borel
1871–1956

The principle that an event with very small probability will not happen is the only law of chance.

- Impossibility on the human scale: $p < 10^{-6}$.
- Impossibility on the terrestrial scale: $p < 10^{-15}$.
- Impossibility on the cosmic scale: $p < 10^{-50}$.

Andrei Kolmogorov
1903–1987

Kolmogorov was credited with establishing measure theory as the mathematical foundation for probability.

Principle A: Over many trials, the frequency with which A happens will approximate P(A).

Principle B: On a single trial, if P(A) very small, we can be practically certain A will not happen.

Jean Ville
1910–1988

Entering the *École Normale Supérieure*.

In 1939, Ville showed that probability can be based on game theory instead of measure theory.

Ville's version of Cournot's principle:

If you bet at fair odds and do not risk bankruptcy, you will not get very rich.

Binary Probability Game

Players: Reality, Skeptic

Protocol:

$\mathcal{K}_0 := 1$.

FOR $n = 1, \dots, N$:

Skeptic announces $s_n \in \mathbb{R}$.

Reality announces $y_n \in \{0, 1\}$.

$\mathcal{K}_n := \mathcal{K}_{n-1} + s_n(y_n - \mathsf{P}\{Y_n = 1 | Y_1 = y_1, \dots, Y_n = y_{n-1}\})$.

Restriction on Skeptic: Skeptic must choose the s_n so that his capital is always nonnegative ($\mathcal{K}_n \geq 0$ for all n) no matter how Reality moves.

Ville showed that Skeptic's getting rich is equivalent to an event of small probability happening:

1. No matter what Skeptic strategy follows,

$$\mathsf{P}\{\mathcal{K}_N \geq \frac{1}{\epsilon}\} \leq \epsilon$$

   for every $\epsilon > 0$.

2. If $\mathsf{P}(A) \leq \epsilon$, then Skeptic has a measurable strategy that guarantees

$$\mathcal{K}_N \geq \frac{1}{\epsilon}$$

   whenever A happens.

The game-theoretic statement of Cournot's principle: You cannot multiply your capital by a large factor without risking bankruptcy.

Vladimir Vovk, born 1960, atop the World Trade Center in 1998.

Classical probability theory generalizes to the case where there are fewer betting offers.

You can still use Cournot's principle to interpret the results!

Probability and Finance: It's Only a Game! Shafer and Vovk, 2001.

www.probabilityandfinance.com

The Market Game

Players: Investor, Market

Protocol:

$\mathcal{K}_0 := 1$.

Market announces $y_0 \in \mathbb{R}$.

FOR $n = 1, 2, \ldots, N$:

  Investor announces $s_n \in \mathbb{R}$.

  Market announces $y_n \in \mathbb{R}$.

  $\mathcal{K}_n := \mathcal{K}_{n-1} + s_n(y_n - y_{n-1})$.

Restriction on Investor: Investor must choose the s_n so that his capital is always nonnegative ($\mathcal{K}_n \geq 0$ for all n) no matter how Market moves.

Example of an implication of Cournot's principle in the market game.

Changes in the price y over an interval of time of length dt scale as $\sqrt{dt}$.

If there are 252 trading days in a year, the typical price change over a year will be

$$\sqrt{252} \approx 16$$

times as large as the typical change over one day.

If this is violated, Investor can make money without risking bankruptcy!!

Defensive forecasting

In 2004, Vovk, Takemura, and Shafer showed how to give forecasts (prices or probabilities) that pass practically any statistical test (strategy for Investor) no matter what the outcomes are.

Protocol:

$\mathcal{K}_0 := 1$.

FOR $n = 1, 2, \ldots, N$:

Opening Market announces $o_n \in \mathbb{R}$. forecast

Investor announces $s_n \in \mathbb{R}$.

Closing Market announces $c_n \in \mathbb{R}$. outcome

$\mathcal{K}_n := \mathcal{K}_{n-1} + s_n(c_n - o_n)$.

Two Kinds of Market Efficiency

A market is speculatively efficient if no investor can multiply their initial capital by a large factor without risking bankruptcy.

A market is informationally efficient if prices are expected values with respect to a probability distribution that is rational (in the sense of Muth, Lucas, and Sargent) and takes all available information into account.

Our defensive forecasting results indicate that speculative efficiency is possible.

The tests markets pass are tests of speculative efficiency.

The bad guy

Joe Doob, 1910–2004, receiving the National Medal of Science from Jimmy Carter in 1979.

Doob put

- continuous random processes
- Ville's idea of a martingale

in the measure-theoretic framework.

In 1965, Samuelson applied Doob's theory of martingales to market prices.

Paul Samuelson
(born 1915)

Present price is the expected value of future price.

It follows that price is a martingale.

The market takes all information into account in calculating the expected value. So if there were good reason to think the price will be different in the future, it would already be different.

For effficency of markets. . .

Burton Malkiel (born 1932)

Malkiel believes the hypothesis has been vindicated by evidence that people cannot make money without undue risk.

Against efficiency of markets. . .

Robert Shiller (born 1946)

Shiller believes the hypothesis has been refuted by evidence that many changes in market prices are not based on new information.

In the game-theoretic framework, Malkiel and Shiller can both be right.

1. Cournot's principle: You will not get rich without risking bankruptcy. As Malkiel points out, the evidence for this remains good.

2. Every change in an asset's price results from new information about the asset. As Shiller points out, there is good evidence against this.