Event trees,
situation spaces,
chain event graphs,
martingale spaces

Conference on reasoning with asymmetric

and context-specific graphs

Baylor University, 19-20 July 2021

https://asym-cntxt-grfs-mtg.netlify.app

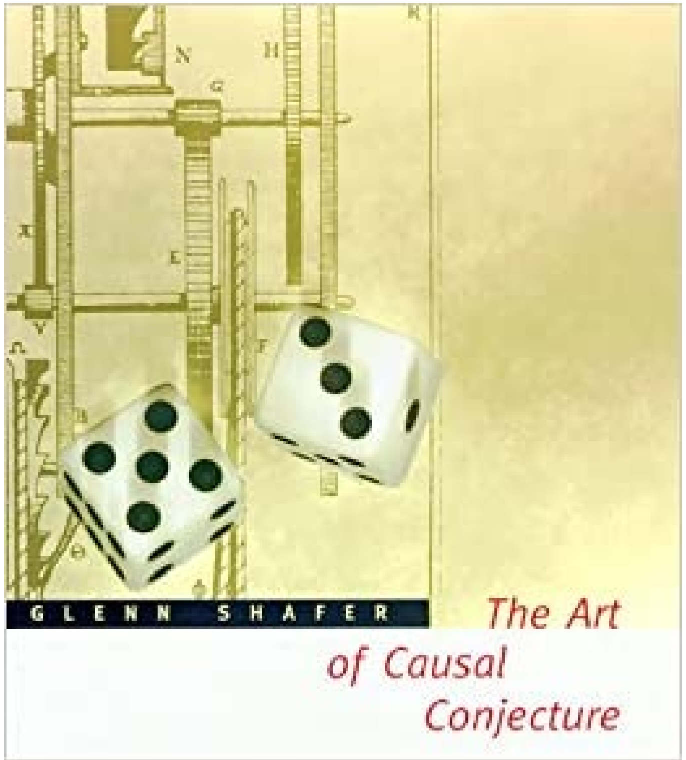
Glenn Shafer, Rutgers University, 19 July 2021

Thank you for this opportunity …

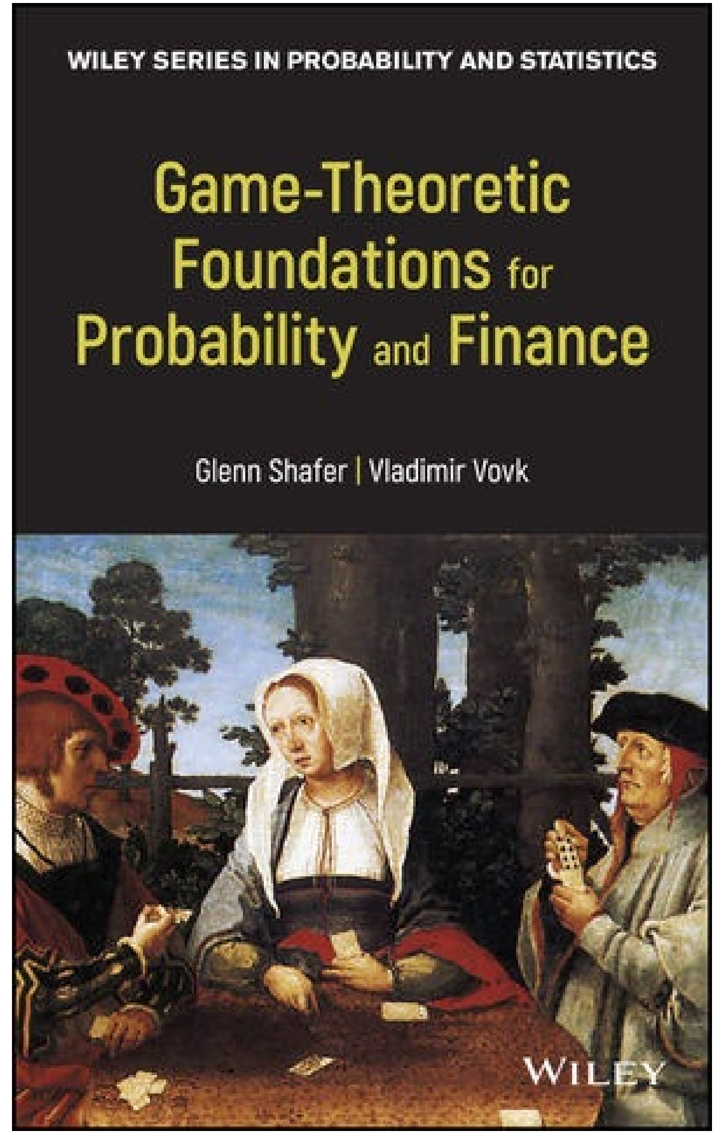
… to re-engage with a topic after 20 years,

… to meet a community.

- [*The Art of Causal Conjecture*] MIT Press 1996.
- [The situation of causality]. *Foundations of Science* **1** 543-563. 1996.
- [Mathematical foundations for probability and causality]. Pp. 207-270 of *Mathematical Aspects of Artificial Intelligence*, edited by Frederick Hoffman. American Mathematical Society, 1998.
- [Causality and responsibility]. *Cardozo Law Review* **22** 101-123. 2001.
- [*Game-Theoretic Foundations for Probability and Finance*] Wiley 2019.


GLENN SHAFER
The Art
of Causal
Conjecture

1996


WILEY SERIES IN PROBABILITY AND STATISTICS
Game-Theoretic
Foundations for
Probability and Finance
Glenn Shafer | Vladimir Vovk
WILEY

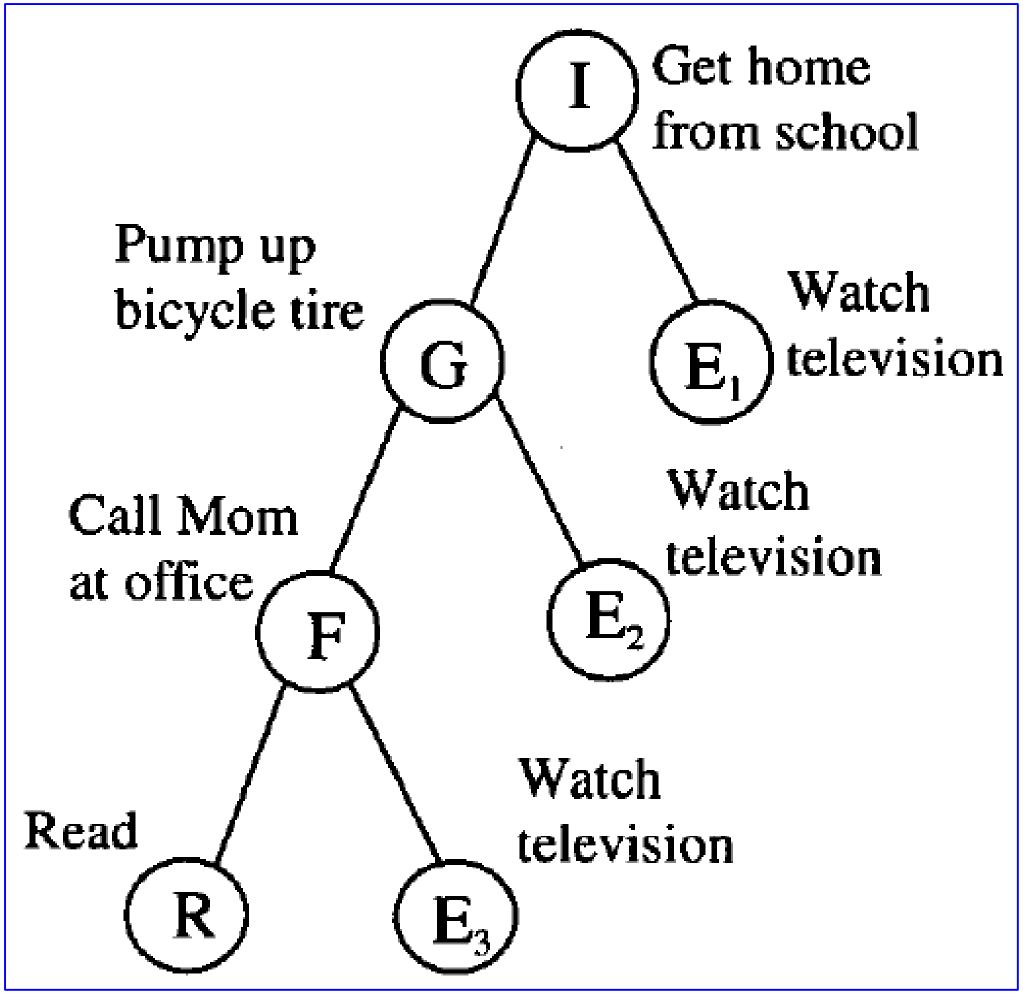
2019

1. Event trees
2. Refinement
3. Relations between instantaneous events (a.k.a situations)
4. Axioms for situation spaces
5. Chain event graphs
6. Predictive causality
7. Supermartingales in situation spaces

1

Event trees

What will Rick do after school?

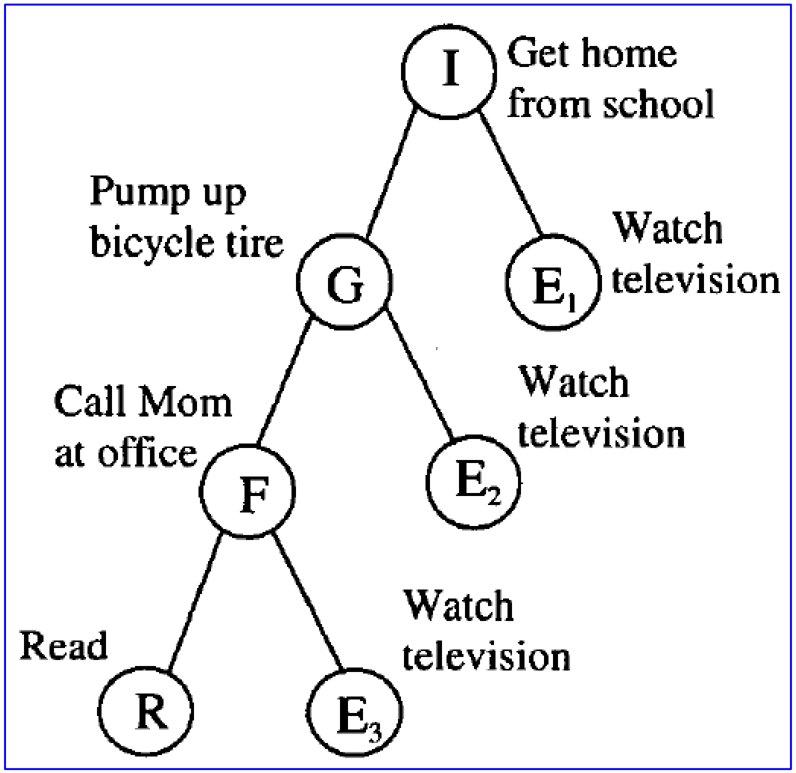


Nodes are **instantaneous events** or **situations**, without duration.

Edges are **actions**, with duration.

The word "event" has too many meanings.

1. Usual meaning in probability: set of root-to-leaf paths, "Moivrean event".
2. Node in the tree: instantaneous event, situation.
3. Step from one node to another: change, action, "Humean event".



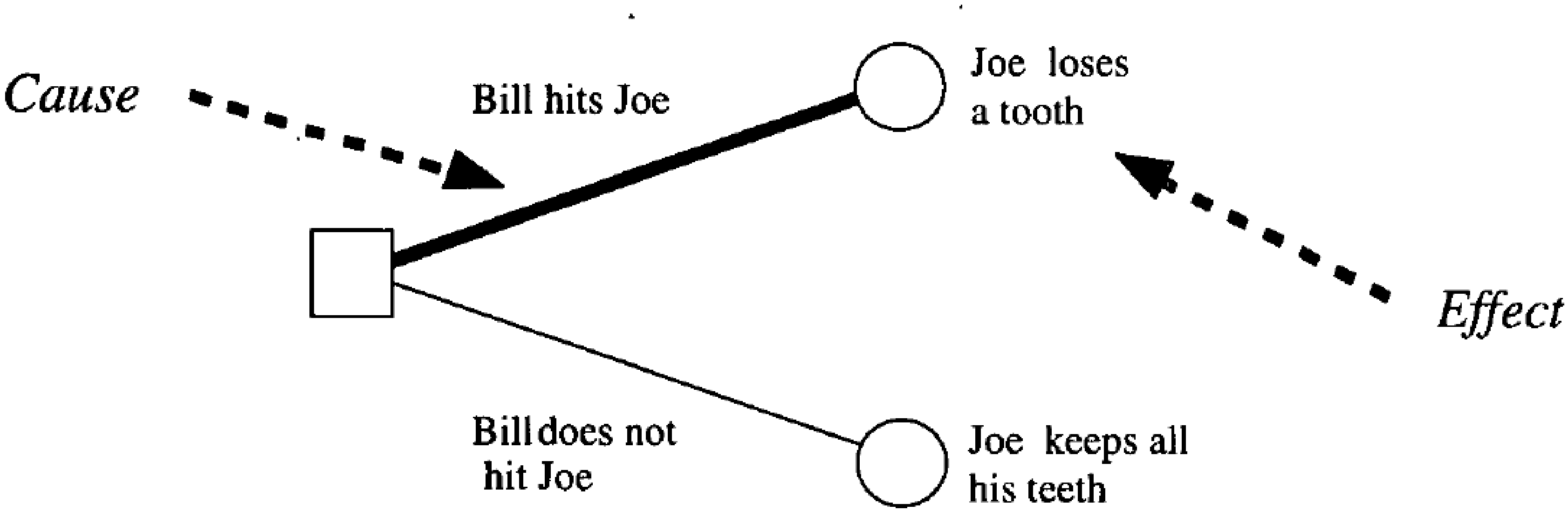
An instantaneous event is not necessarily labeled with a single clock time.

Rick intends to pump up his bicycle time when he gets home, but he will eat a cookie first, and at some point he may get distracted and start watching television.

How should we think about cause and effect in an event tree?

In my 2001 *Cardozo Law Review* paper, I suggested that

- cause = step in the tree (Humean event);
- effect = node in the tree (situation).



Axioms for finite event trees

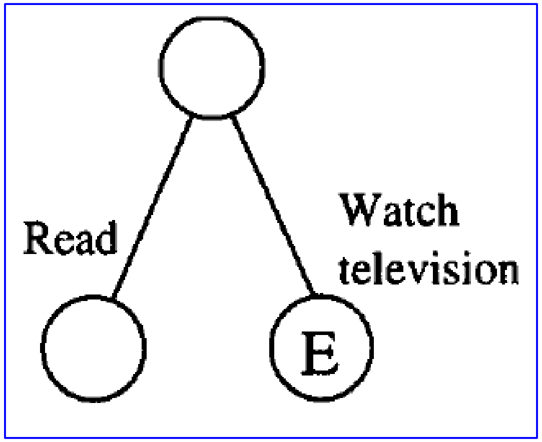
A finite event tree is a finite set with a partial ordering *precedes* such that:

- There is an element that precedes all the others.

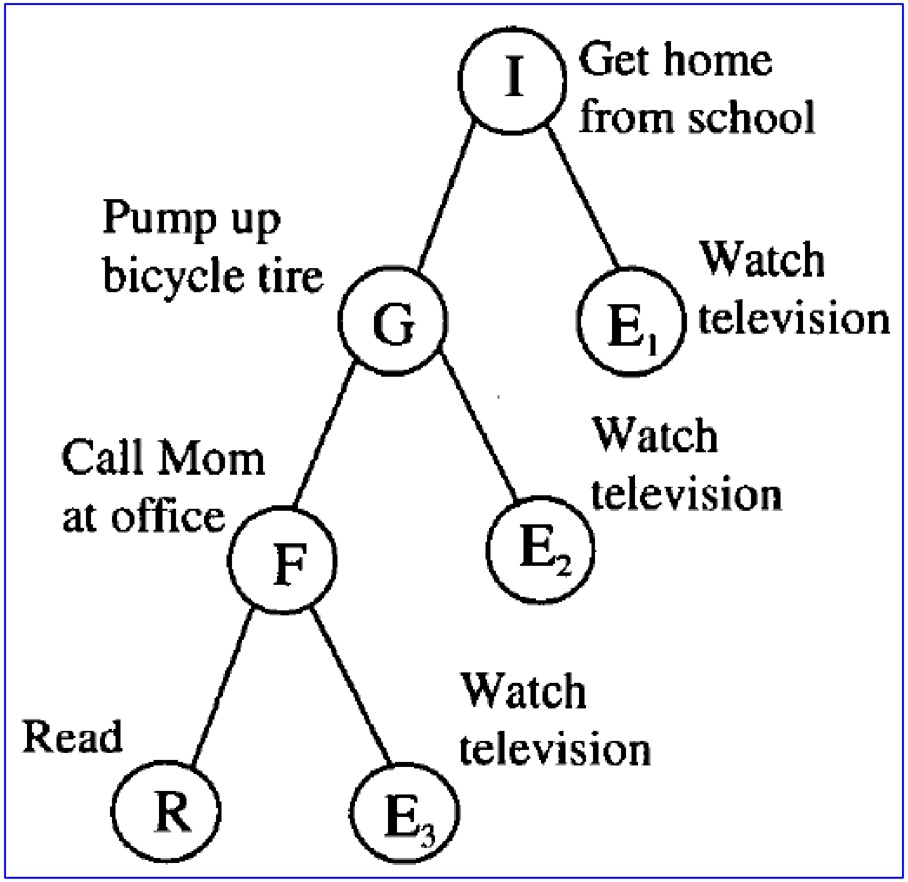
- If S and T both precede U, then either S precedes T or T precedes S.

2

Refinement

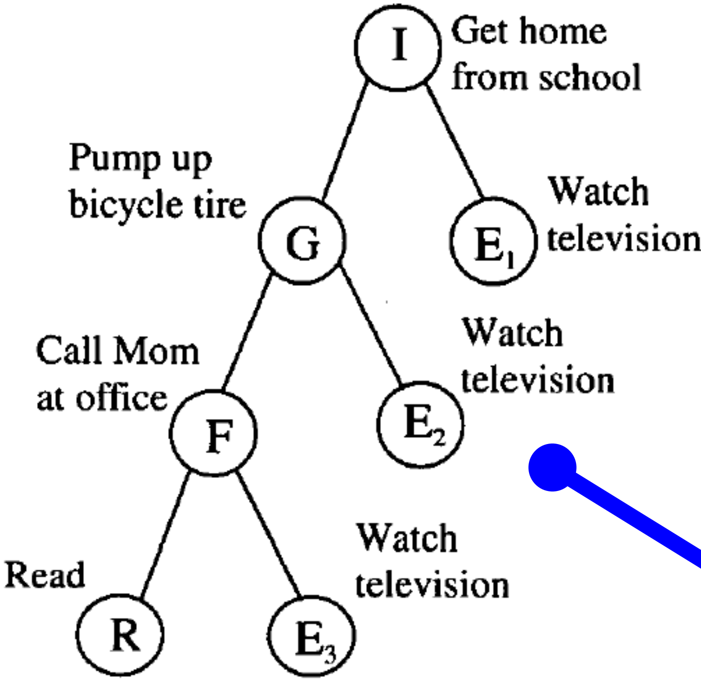


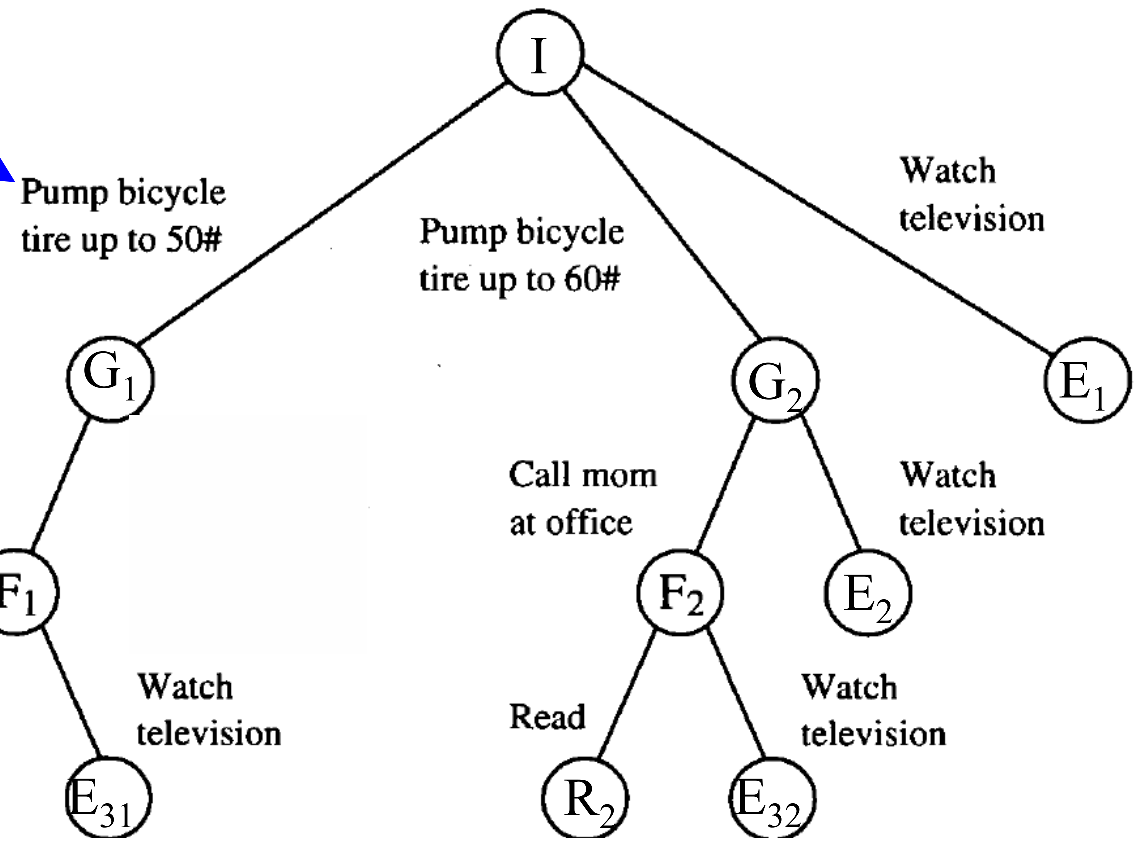
Refine



So E_1, E_2, and E_3 can be thought of as a single situation.

Refine further



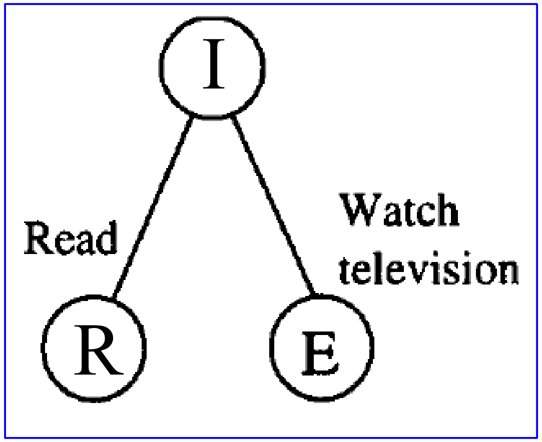


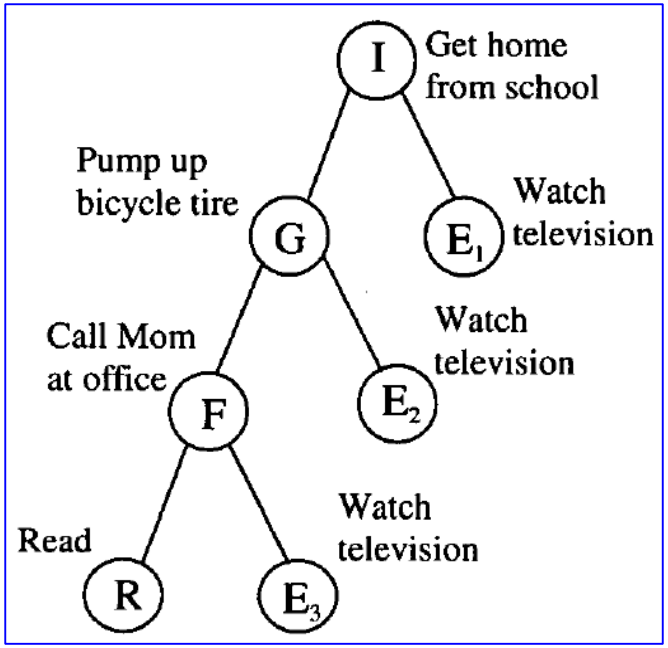
3

Relations between *instantaneous events* (a.k.a. *situations*)

Consider *all* the situations in our story. This includes I, G, F, R, E, and E_1, E_2, and E_3.

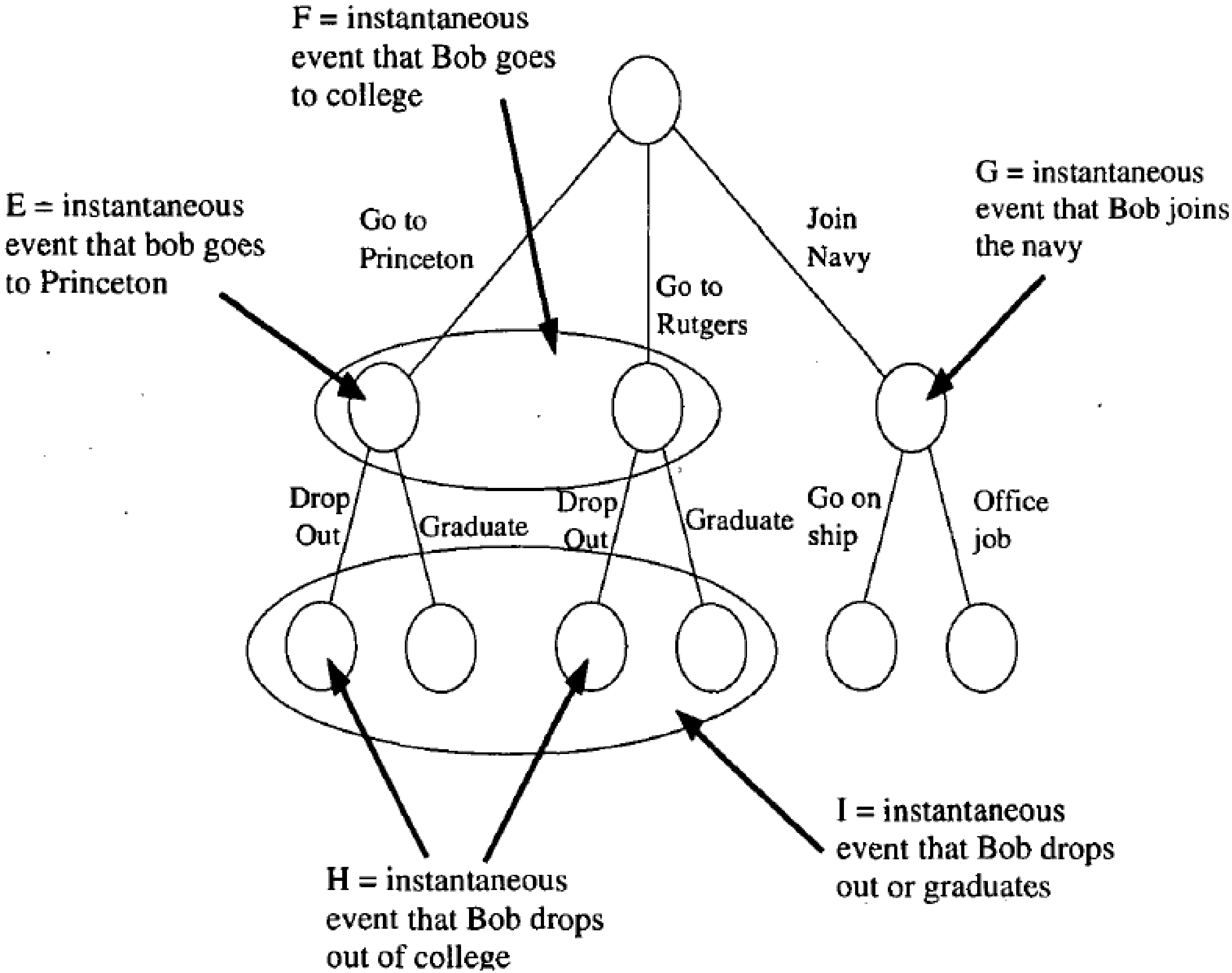
Also the situation obtained by merging F and E_2, the situation obtained by merging F and E_3, etc.

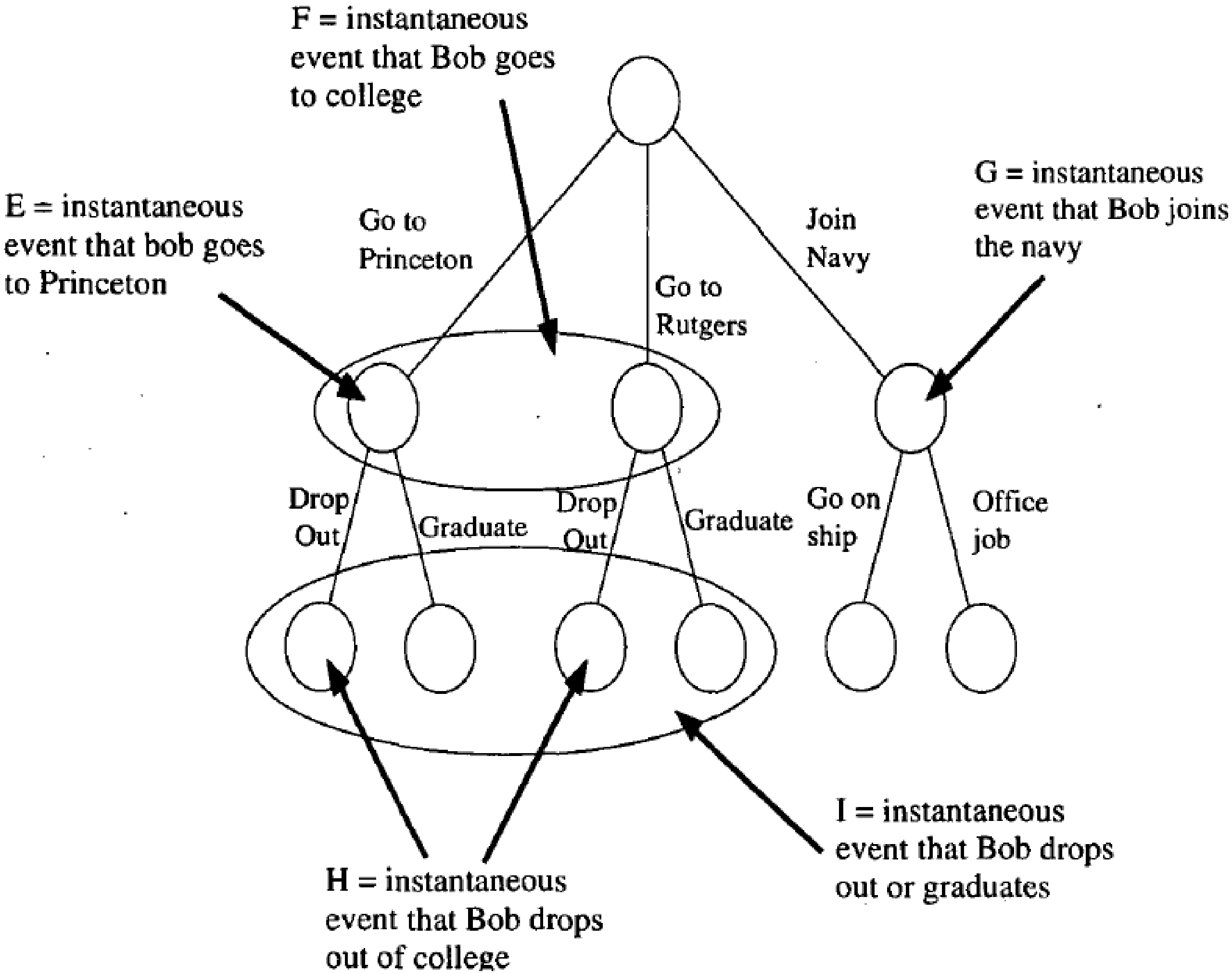




What are the (binary) relations among these situations, and how can we axiomatize them?

Are **precedes** and **refines** enough? No.


F = instantaneous event that Bob goes to college
E = instantaneous event that bob goes to Princeton
Go to Princeton
Go to Rutgers
Join Navy
G = instantaneous event that Bob joins the navy
Drop Out
Graduate
Drop Out
Graduate
Go on ship
Office job
H = instantaneous event that Bob drops out of college
I = instantaneous event that Bob drops out or graduates



E *allows* H.	No matter how E happens, H can happen later.
H *requires* F.	No matter how H happens, F has already happened.
F *precedes* H.	F allows H and H requires F.
E *foretells* I.	When E happens, I's later happening is inevitable.
F *always foretells* I.	F foretells I and I requires F.

4

Axioms for situation spaces

In my 1998 “Mathematical foundations for probability and causality”, I considered how the notion of an “instantaneous event space” or “situation space” can be axiomatized.

I found that I needed 4 or 5 binary relations.

PRIMARY MANDATORY RELATIONS

S **refines** T	Whenever S happens, T happens at the same time.
S **str.requires** T	Whenever S happens, T has already happened.
S **str.foretells** T	Whenever S happens, T must happen later.
S **forbears** T	Whenever S happens, T remains only possible.
S **divergent** T	Whenever S happens, T cannot happen.

My axioms were complex, and I don't think anyone followed up with them.

Perhaps I was misguided to insist on <u>binary</u> relations.

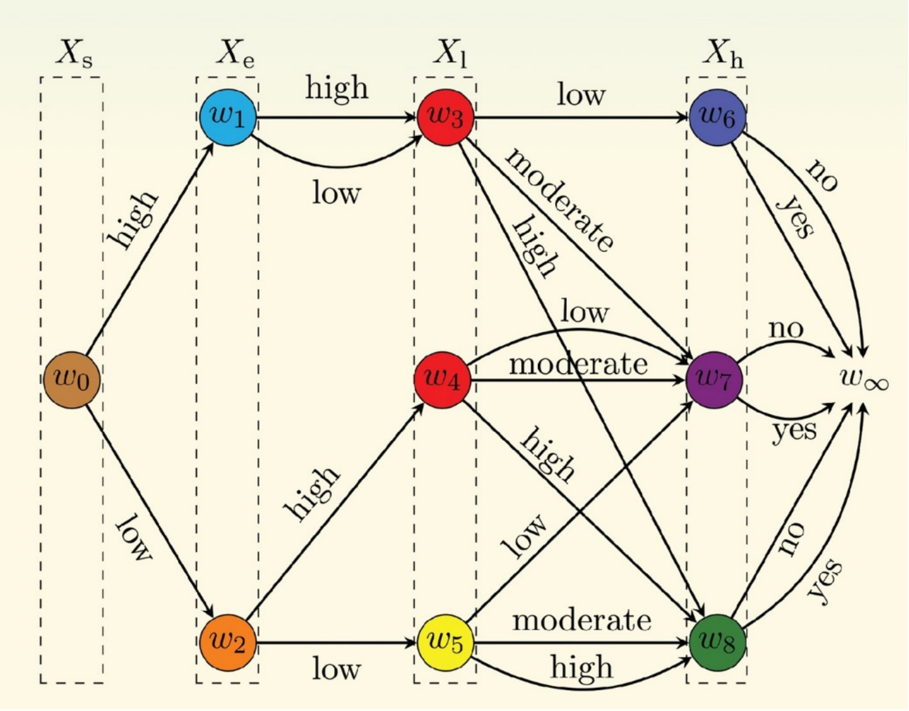
Perhaps an adequate axiomatization can be based on these four relations:

- situation S precedes situation T,
- situation S implies situation T,
- situation S diverges from situation T,
- set of situations $\mathcal{C}$ composes situation T.

5

Chain event graphs

Only some of the situations in the situation space are shown.



6

Predictive causality

According to this philosophy, causal structure is the structure of predictions by a superior intelligence, who witnesses everything that could conceivably be witnessed by a human-like witness and predicts everything that could conceivably be predicted by a human-like scientist.

I call this superior intelligence "Nature."

Like us and unlike God, Nature moves through time.

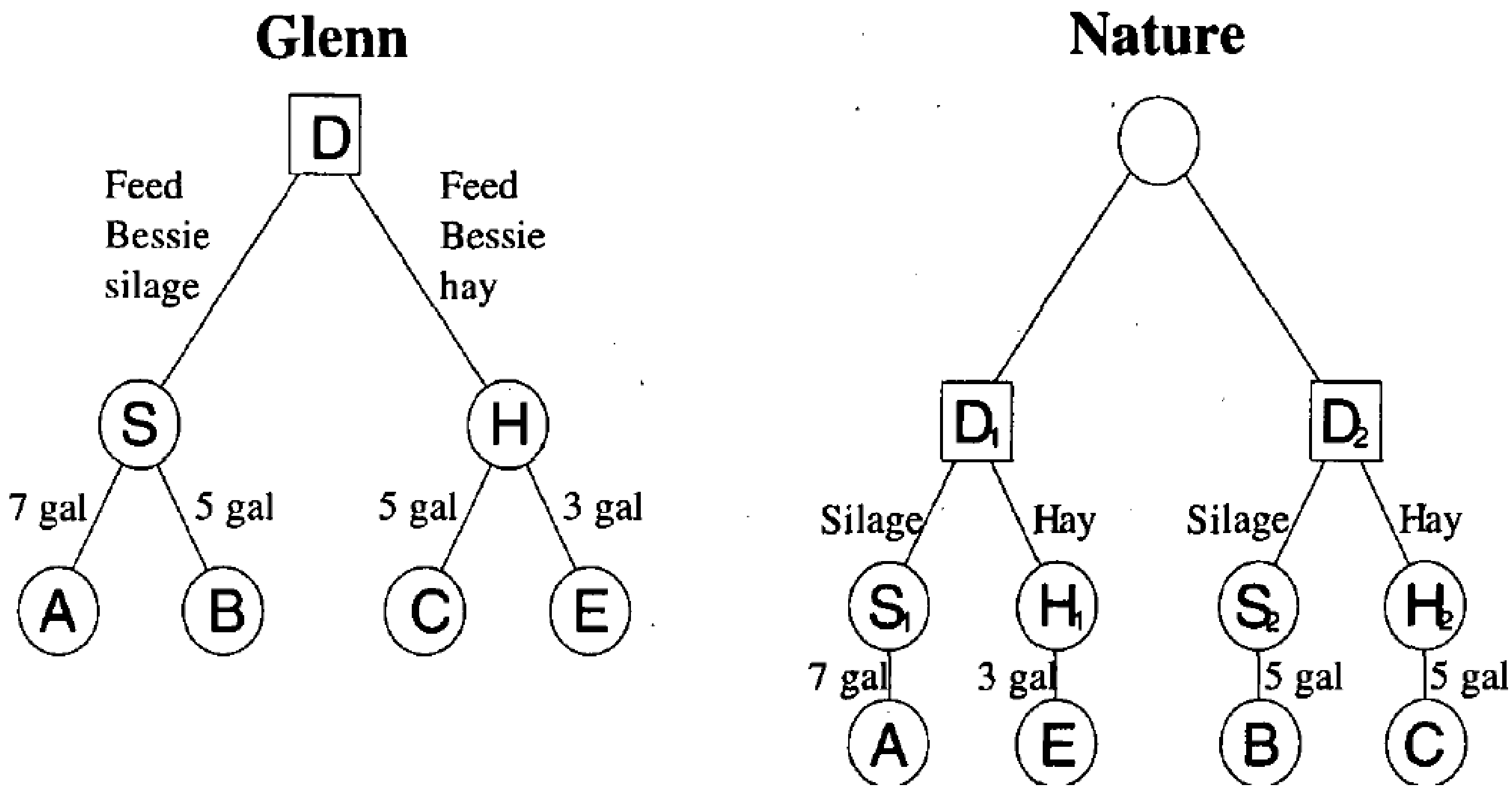
Her knowledge increases as she sees what happens.

Like us, Nature can predict some events in advance but cannot predict everything.

Our knowledge can be empirically valid but still not causal.

Nature's superior knowledge may allow her to rule out some things we regard as possible but will not, if our knowledge is valid, allow her to rule in things we know to be impossible.

Glenn knows less than Nature.



Doob's fundamental idea in martingale theory: Use notion of filtration to express that you are not a prophet.

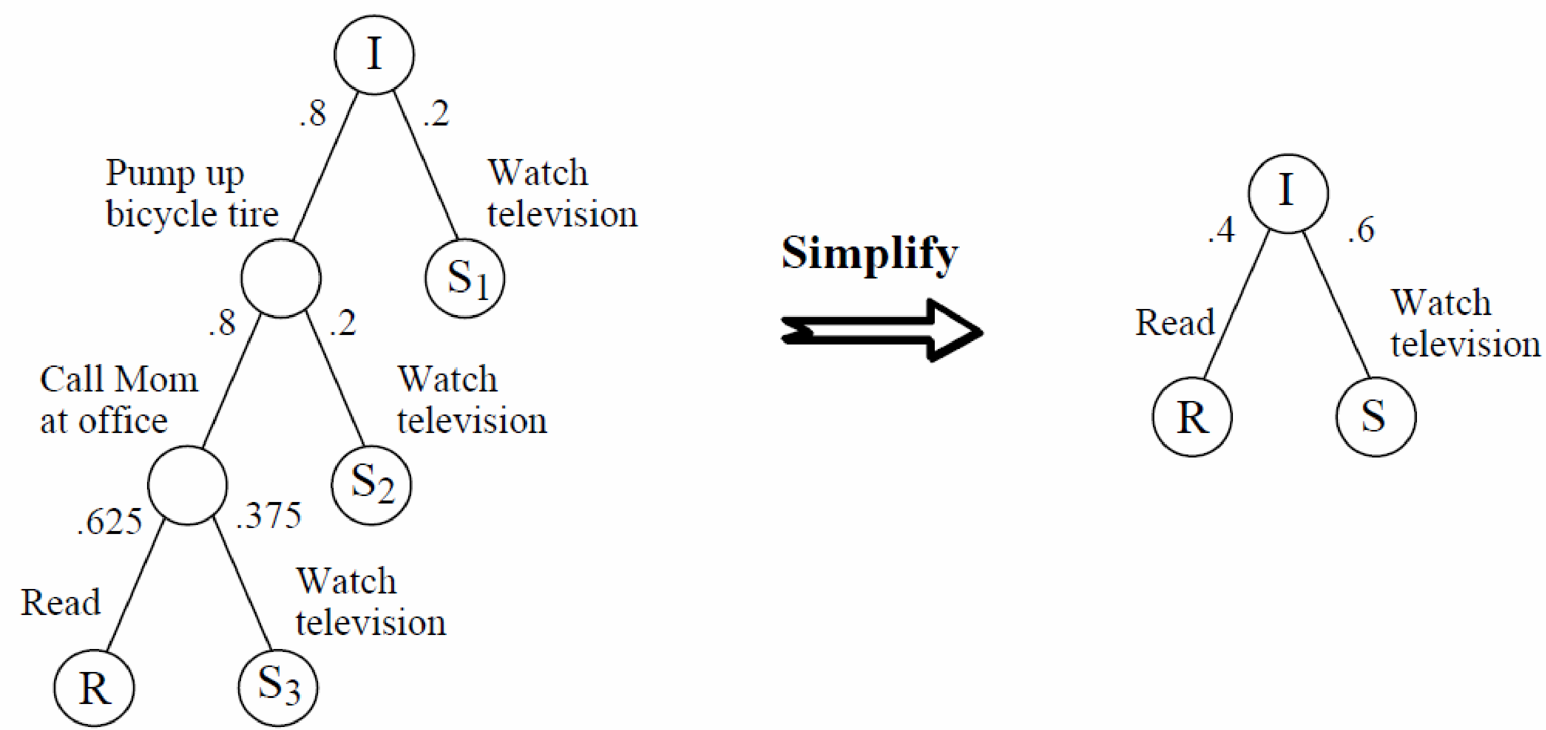
You have a filtration. Prophet has a larger filtration, because he has more knowledge at every moment.

Theorem: Martingales in small filtration become semimartingales in large filtration, where they decompose into "signal plus noise".

Barlow 1978, Yor and de Sam Lazaro 1978, Jeulin 1980. Cited by Paul-André Meyer: Stochastic processes from 1950 to the present jehps.net, June 2009, as chapter in the general theory of processes.

Probability in situation spaces should be richer than measure theory with multiple filtrations.

- No filtration-like time scale.
- Allow decision causes (steps).
- Allow causes with upper and lower probabilities.


I
.8
.2
Pump up
bicycle tire
Watch
television
S1
.8
.2
Call Mom
at office
Watch
television
S2
.625
.375
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television
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.4
.6
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television
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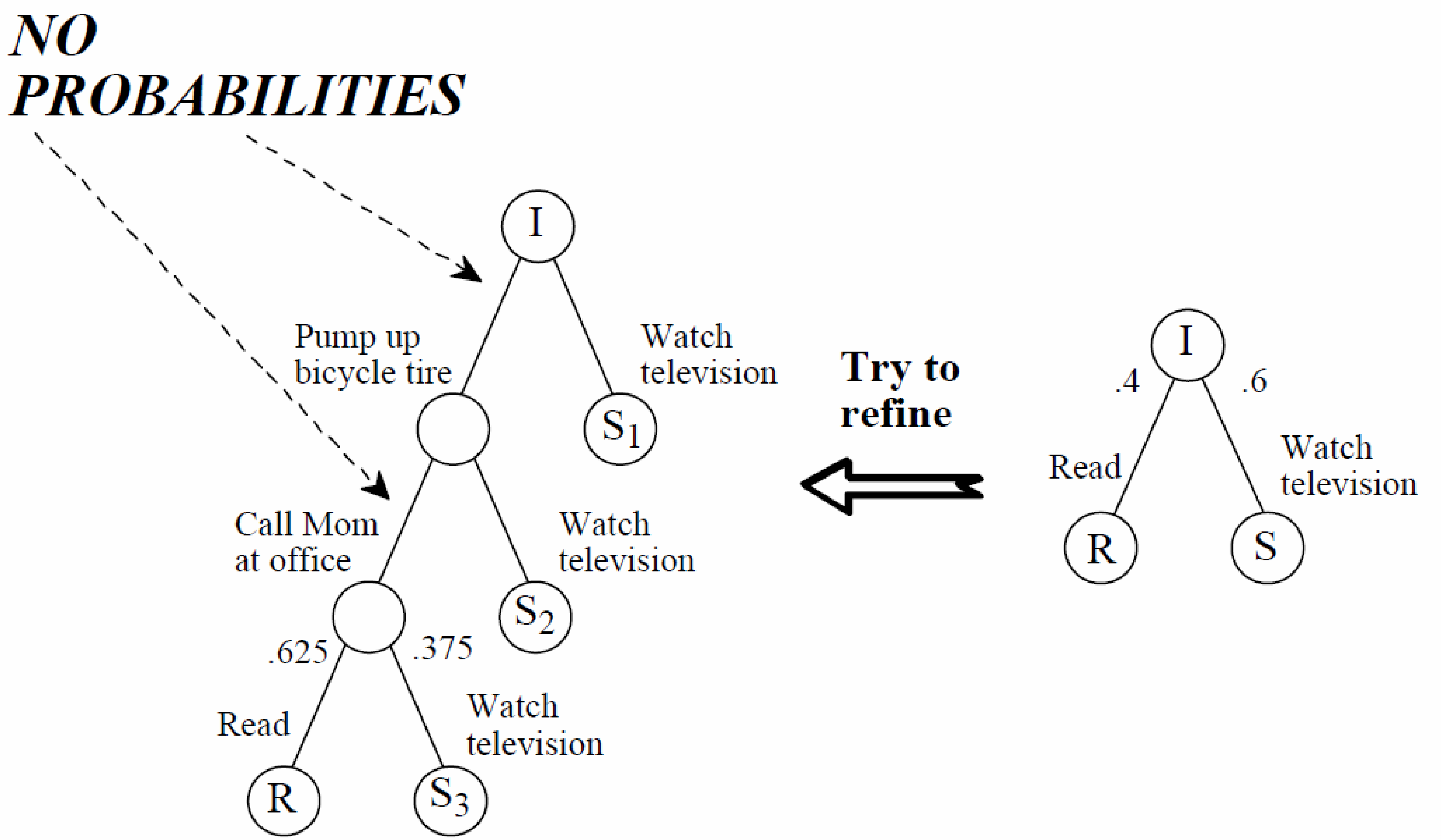
NO
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Pump up
bicycle tire
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television
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Call Mom
at office
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.375
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refine
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.4
.6
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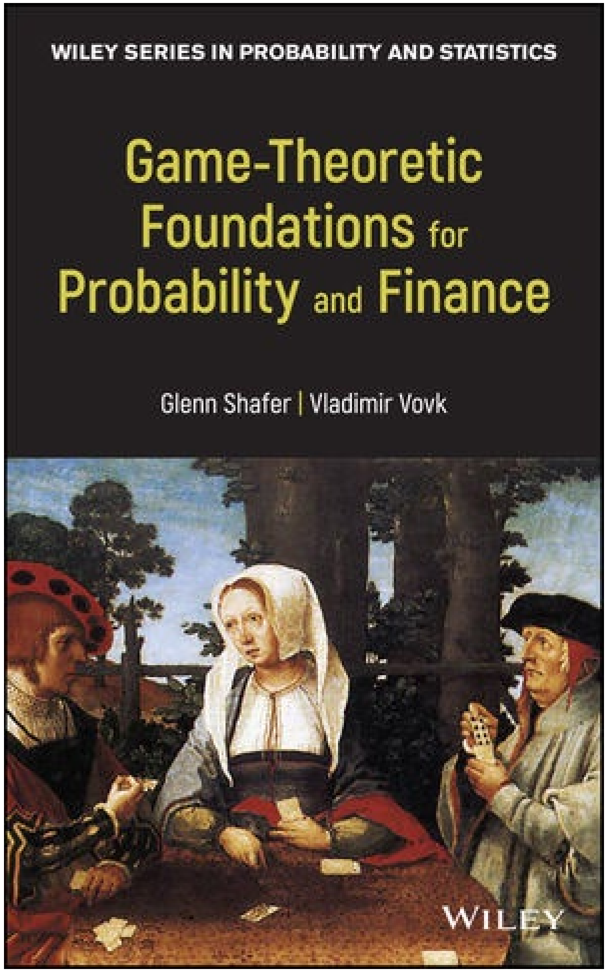
Watch
television
R
S

7

Supermartingales in situation spaces



Generalizes probability theory to upper and lower probability in event trees.

Forecasting strategy: offers gambles in each situation.

Gambling strategy: chooses a gamble in each situation.

Supermartingale: resulting capital process (assigns $ amount to each situation)

Upper probability of E: amount gambler must risk to win one unit if *E* happens.

A payoff g is an extended real-valued function on the children.

An offer is a set $\mathbf{G}$ of payoffs.

Axiom G1. If $g_1, g_2 \in \mathbf{G}$, then $g_1 + g_2 \in \mathbf{G}$.

Axiom G2. If $c \in [0, \infty)$ and $g \in \mathbf{G}$, then $cg \in \mathbf{G}$.

Axiom G3. If $g_1 \in \overline{\mathbb{R}}^{\mathcal{Y}}$, $g_2 \in \mathbf{G}$, and $g_1 \leq g_2$, then $g_1 \in \mathbf{G}$.

Axiom G4. If $g \in \mathbf{G}$, then $\inf g \leq 0$.

How do we generalize further to situation spaces?

Forecasting strategy: offers gambles in each situation. Payoffs of an offer can be in arbitrary descendants, not just in children.

Gambling strategy: chooses a gamble in each situation.

Supermartingale: resulting capital process (assigns $ amount to each situation)

Upper probability of E: amount gambler must risk to win one unit if *E* happens.